Implementation of a new point reactor kinetics calculation strategy for MSR neutron dynamics in the system-code CATHARE

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Preamble

HOFSTADTER's Law: It always takes longer than you expect, even when you take into account HOFSTADTER's Law.

GÖDEL, ESCHER, BACH: an eternal golden braid.
Douglas R. HOFSTADTER, 1999.

I. **Neutron importance function, adjoint flux**

II. Point reactor kinetics model for MSRs

Neutron importance function

Definition

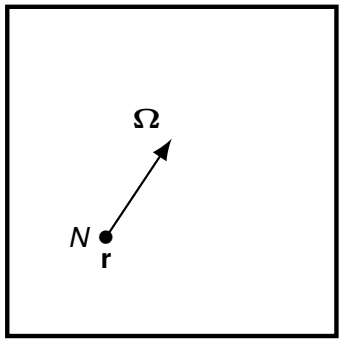
We consider a classical time-dependent neutron transport problem:

$$\begin{cases} \dfrac{1}{v}\dfrac{\partial \psi}{\partial t} = -\boldsymbol{\Omega}\cdot\nabla\psi - \Sigma_t\psi + \displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \Sigma_s\left(\mathbf{r}, E' \to E, \boldsymbol{\Omega}' \to \boldsymbol{\Omega}, t\right)\psi(\mathbf{r}, E', \boldsymbol{\Omega}', t) \\ \qquad + \dfrac{\chi_p(\mathbf{r}, E)}{4\pi}\displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E'(1-\beta)\nu\Sigma_f(\mathbf{r}, E', t)\int_{\mathbb{S}_2}\mathrm{d}\Omega'\psi(\mathbf{r}, E', \boldsymbol{\Omega}', t) + \displaystyle\sum_{i=1}^{F}\frac{\chi_{d,i}(\mathbf{r}, E)}{4\pi}\lambda_i c_i(\mathbf{r}, t) \\ \dfrac{\partial c_i}{\partial t} = -\lambda_i c_i + \displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E'\beta_i\nu\Sigma_f(\mathbf{r}, E', t)\int_{\mathbb{S}_2}\mathrm{d}\Omega'\psi(\mathbf{r}, E', \boldsymbol{\Omega}', t) \quad , \qquad 1 \le i \le F \\ + \text{ boundary conditions } + \text{ initial conditions} \end{cases}$$

In general, it does not admit any non-trivial stationary solution, unless we divide ν by the reactor k_{eff} for a given and fixed set of operating parameters.

Let's consider the reactor made critical by the division of ν by k_{eff} and such that ψ and the c_i are null at time 0, and let's inject N neutrons of energy E and direction $\boldsymbol{\Omega}$ at point $\mathbf{r}$.

The reactor being critical, after a transient period, these neutrons will eventually establish an equilibrium flux characterized by a certain power level P, which depends on $\mathbf{r}$, E, and $\boldsymbol{\Omega}$, and which is proportional to N.



We define the neutron importance function for neutrons of energy E, direction $\boldsymbol{\Omega}$, positioned in $\mathbf{r}$, as: $\psi^*(\mathbf{r}, E, \boldsymbol{\Omega}) := \text{constant} \times P/N$

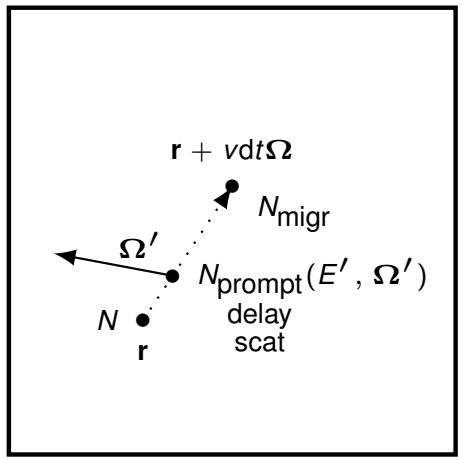
The neutron importance function satisfies an integro-differential equation which describes the balance summarized by the following statement:

> *The total importance of the neutrons ensuing from the N initial neutrons after an infinitesimal duration dt is equal to the total importance of the N initial neutrons.*

Let's make this balance. During dt, among the N initial neutrons, N_{inter} will interact with the isotopes of the reactor materials, while the remaining N_{migr} will migrate without interacting until reaching the position $\mathbf{r} + v\,dt\,\mathbf{\Omega}$.

Some of these interactions will be collisions. They will give birth to $N_{\text{scat}}(E', \mathbf{\Omega}')$ neutrons of energy E' and direction $\mathbf{\Omega}'$.

Another part will be fissions. There will be N_{fiss} of them. They will give birth to $N_{\text{prompt}}(E', \mathbf{\Omega}')$ prompt neutrons of energy E' and direction $\mathbf{\Omega}'$, and $N_{\text{prec},i}$ delayed neutron precursors of the i-th group. *Since they will not migrate*, all the neutrons they will eventually emit will be emitted from $\mathbf{r}$. There will be $N_{\text{delay},i}(E', \mathbf{\Omega}')$ with energy E' and direction $\mathbf{\Omega}'$.



$$
\begin{cases}
N_{\text{inter}} \equiv N\,v\,\mathrm{d}t\,\Sigma_t \qquad N_{\text{migr}} \equiv N\,(1 - v\,\mathrm{d}t\,\Sigma_t) \\
N_{\text{scat}}(E', \mathbf{\Omega}') \equiv N\,v\,\mathrm{d}t\,\Sigma_s(\mathbf{r}, E \to E', \mathbf{\Omega} \to \mathbf{\Omega}') \\
N_{\text{fiss}} \equiv N\,v\,\mathrm{d}t\,\Sigma_f(\mathbf{r}, E) \qquad N_{\text{prec},i} \equiv \beta_i\,\nu/k_{\text{eff}}\,N_{\text{fiss}} \\
N_{\text{prompt}}(E', \mathbf{\Omega}') \equiv \chi p(\mathbf{r}, E')/4\pi\,(1 - \beta)\,\nu/k_{\text{eff}}\,N_{\text{fiss}} \\
N_{\text{delay},i}(E', \mathbf{\Omega}') \equiv \chi_{d,i}(\mathbf{r}, E')/4\pi\,\beta_i\,\nu/k_{\text{eff}}\,N_{\text{fiss}}
\end{cases}
$$

The balance equation thus reads:

$$N \times \psi^*(\mathbf{r}, E, \mathbf{\Omega}) = N_{\text{migr}} \times \psi^*(\mathbf{r} + v\mathrm{d}t\mathbf{\Omega}, E, \mathbf{\Omega}) + \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \left[N_{\text{scat}}(E', \mathbf{\Omega}') + N_{\text{prompt}}(E', \mathbf{\Omega}') + \sum_{i=1}^{F} N_{\text{delay},i}(E', \mathbf{\Omega}') \right] \times \psi^*(\mathbf{r}, E', \mathbf{\Omega}')$$

Which, with some algebra, and letting $dt \to 0$, leads to:

$$0 = \mathbf{\Omega} \cdot \nabla\psi^*(\mathbf{r}, E, \mathbf{\Omega}) - \Sigma_t(\mathbf{r}, E)\psi^*(\mathbf{r}, E, \mathbf{\Omega}) + \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \Sigma_s\left(\mathbf{r}, E \to E', \mathbf{\Omega} \to \mathbf{\Omega}'\right) \psi^*(\mathbf{r}, E', \mathbf{\Omega}') + \frac{1}{k_{\text{eff}}} \left[(1-\beta)\nu\Sigma_f(\mathbf{r}, E) \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \frac{\chi_p(\mathbf{r}, E')}{4\pi} \psi^*(\mathbf{r}, E', \mathbf{\Omega}') + \sum_{i=1}^{F} \beta_i \nu\Sigma_f(\mathbf{r}, E) \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \frac{\chi_{d,i}(\mathbf{r}, E')}{4\pi} \psi^*(\mathbf{r}, E', \mathbf{\Omega}') \right]$$

Neutron importance function

Adjoint flux

It turns out that the operator $\mathcal{A}^*$ involved in this equation is the adjoint operator of the operator $\mathcal{A}$ involved in the stationary eigen-equation bearing on ψ, for the scalar product:

$$\langle f, g \rangle := \int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, f(\mathbf{r}, E, \mathbf{\Omega}) \, \overline{g(\mathbf{r}, E, \mathbf{\Omega})}$$

This is to say: $\langle \mathcal{A}f, g \rangle \equiv \langle f, \mathcal{A}^* g \rangle \; \forall f, g$.

For this reason, the neutron importance function is often called “adjoint flux”.

Neutron importance function for MSRs

We now consider the time-dependent neutron transport problem under fuel circulation conditions:

$$
\begin{cases}
\dfrac{1}{v}\dfrac{\partial \psi}{\partial t} = -\boldsymbol{\Omega}\cdot\nabla\psi - \Sigma_t\psi + \displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \Sigma_s\left(\mathbf{r}, E' \to E, \boldsymbol{\Omega}' \to \boldsymbol{\Omega}, t\right)\psi(\mathbf{r}, E', \boldsymbol{\Omega}', t) \\
\qquad + \dfrac{\chi_p(\mathbf{r}, E)}{4\pi}\displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega'(1-\beta)\nu\Sigma_f(\mathbf{r}, E', t)\psi(\mathbf{r}, E', \boldsymbol{\Omega}', t) + \displaystyle\sum_{i=1}^{F} \dfrac{\chi_{d,i}(\mathbf{r}, E)}{4\pi}\lambda_i c_i(\mathbf{r}, t) \\
\dfrac{\partial c_i}{\partial t} - \dfrac{\boldsymbol{\nu_s}}{\mathbf{Sc}}\boldsymbol{\Delta c_i} + \boldsymbol{\nabla\cdot(c_i \mathbf{v})} + \lambda_i c_i = \displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega'\, \beta_i\, \nu\Sigma_f(\mathbf{r}, E', t)\, \psi(\mathbf{r}, E', \boldsymbol{\Omega}', t)\ ,\ 1 \le i \le F \\
+ \text{ CFD equations } + \text{ boundary conditions } + \text{ initial conditions}
\end{cases}
$$

Just as in the classical case, if we consider the reactor operating parameters at any given time t to be fixed, there generally exists no non-trivial stationary neutron flux and delayed neutron precursor concentrations solution to the problem.

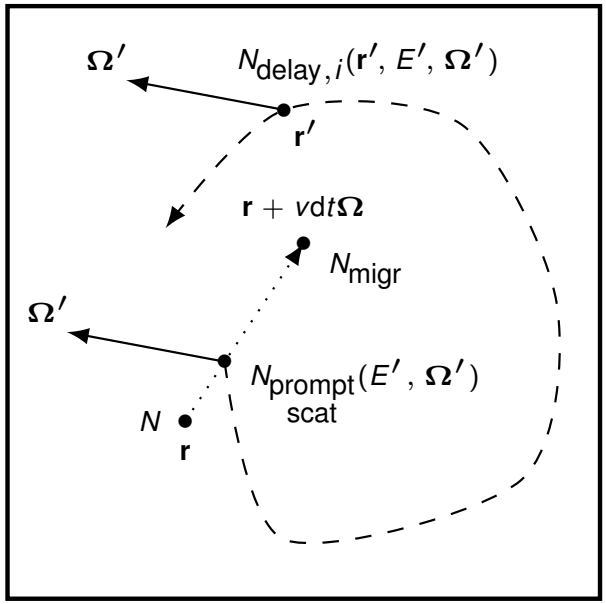
However, it is still possible to find a k_{eff} making the reactor critical, and enabling the existence of a stationary neutron flux and precursor concentrations.

Neutron importance function for MSRs

Just as in the classical case, let's consider the reactor made critical and such that ψ and the c_i are null at time 0; let's inject N neutrons of energy E and direction $\mathbf{\Omega}$ at point $\mathbf{r}$; and let's inventory the neutrons ensuing from the N initial neutrons after an infinitesimal duration dt.

As in the classical case:

- N_{migr} will migrate without interacting until reaching the position $\mathbf{r} + v\,\mathrm{d}t\,\mathbf{\Omega}$;
- $N_{\text{scat}}(E', \mathbf{\Omega}')$ of energy E' and direction $\mathbf{\Omega}'$ will emerge from collisions;
- $N_{\text{prompt}}(E', \mathbf{\Omega}')$ prompt neutrons of energy E' and direction $\mathbf{\Omega}'$ and $N_{\text{prec},i}$ delayed neutron precursors of the i-th group will emerge from fissions.



However, contrary to the classical case, the $N_{prec,i}$ precursors will migrate in the fuel circuit, and they will emit neutrons in a spatially-distributed manner.

All calculus being done, and letting $dt \to 0$, we find:

Neutron importance function for MSRs

$$
\begin{cases}
0 = \boldsymbol{\Omega}\cdot\nabla\psi^*(\mathbf{r},E,\boldsymbol{\Omega}) - \Sigma_t(\mathbf{r},E)\psi^*(\mathbf{r},E,\boldsymbol{\Omega}) \\
\qquad + \displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \Sigma_s\left(\mathbf{r}, E\to E', \boldsymbol{\Omega}\to\boldsymbol{\Omega}'\right)\psi^*(\mathbf{r},E',\boldsymbol{\Omega}') \\
\qquad + \dfrac{1}{k_{\text{eff}}}\nu\Sigma_f(\mathbf{r},E)\left[(1-\beta)\displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \frac{\chi_p(\mathbf{r},E')}{4\pi}\psi^*(\mathbf{r},E',\boldsymbol{\Omega}') + \sum_{i=1}^{F}\lambda_i c_i^*(\mathbf{r})\right] \\
\\
-\dfrac{\nu_s}{\mathrm{Sc}}\Delta c_i^* - \nabla\cdot(c_i^*\mathbf{v}) + \lambda_i c_i^* = \beta_i \displaystyle\int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \frac{\chi_{d,i}(\mathbf{r}',E')}{4\pi}\psi^*(\mathbf{r}',E',\boldsymbol{\Omega}') \\
\\
+ \text{ boundary conditions}
\end{cases}
$$

where we have introduced the “adjoint precursor concentrations” c_i^*.

(This derivation and expression is similar to that presented in [Aufiero et al. 2014].)

We will now see that the neutron importance function can be used to derive a point reactor kinetics model appropriate for MSRs.

I. Neutron importance function, adjoint flux

II. Point reactor kinetics model for MSRs

Derivation of a PRK model for MSRs

Flux factorization and amplitude equation

As is usually done in the classical case, the neutron flux is factored as:

$$\psi(\mathbf{r}, E, \mathbf{\Omega}, t) := a(t) \times f(\mathbf{r}, E, \mathbf{\Omega}, t)$$

This factorization is unique if we impose the additional condition:

$$\left\langle \psi_0^*, \frac{1}{v} f(\cdot, t) \right\rangle := \int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \psi_0^*(\mathbf{r}, E, \mathbf{\Omega}) \cdot \frac{1}{v} f(\mathbf{r}, E, \mathbf{\Omega}, t) \equiv c \quad \forall t$$

where ψ_0^* is the adjoint flux ($\Leftrightarrow$ neutron importance function) at time 0, and c an arbitrary constant.

Injecting this factorization in Boltzmann's equation, multiplying the equation by ψ_0^*, integrating it with respect to $\mathbf{r}$, E and $\mathbf{\Omega}$, and accounting for the previous condition, we obtain:

$$\frac{\mathrm{d}a}{\mathrm{d}t} = \left(\frac{\rho(t)}{\Lambda(t)} - \sum_{i=1}^{F} \frac{\widetilde{\beta}_i(t)}{\Lambda(t)} \right) a(t) + \sum_{i=1}^{F} \lambda_i \widetilde{c}_i(t)$$

where the introduced parameters have the following definitions:

Derivation of a PRK model for MSRs

Kinetic parameters definitions

- $\Lambda(t) := \dfrac{\mathcal{I}^0(t)}{\mathcal{I}_f^0(t)}$ is the “effective generation time”;

- $\mathcal{I}^0(t) := \displaystyle\int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \frac{1}{v} f(\mathbf{r}, E, \mathbf{\Omega}, t) \psi_0^*(\mathbf{r}, E, \mathbf{\Omega})$
  is “the total importance of all neutrons of the reactor”;

- $\mathcal{I}_f^0(t) := \displaystyle\int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \nu\Sigma_f(\mathbf{r}, E, t) f(\mathbf{r}, E, \mathbf{\Omega}, t)$
  $\displaystyle\times \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \, \frac{(1-\beta)\chi_p(\mathbf{r}, E') + \sum_1^F \beta_i \chi_{d,i}(\mathbf{r}, E')}{4\pi} \psi_0^*(\mathbf{r}, E', \mathbf{\Omega}')$
  is “the total importance of the neutrons produced by fission per unit time”;

- $\rho(t) := \dfrac{\mathcal{I}_b^0(t)}{\mathcal{I}_f^0(t)}$ is the “dynamic reactivity”;

- $\mathcal{I}_b^0(t) := -\mathcal{I}_d^0(t) + \mathcal{I}_c^0(t) + \mathcal{I}_f^0(t)$ is “the total algebraic importance of the neutrons being produced and disappearing per unit time”;

Derivation of a PRK model for MSRs

Kinetic parameters definitions

- $\mathcal{I}_d^0(t) := -\int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \left(-\mathbf{\Omega} \cdot \nabla f(\mathbf{r}, E, \mathbf{\Omega}, t) - \Sigma_t(\mathbf{r}, E, t) f(\mathbf{r}, E, \mathbf{\Omega}, t) \right) \psi_0^*(\mathbf{r}, E, \mathbf{\Omega})$

  is "the total importance of the neutrons disappearing per unit time";

- $\mathcal{I}_c^0(t) := \int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \psi_0^*(\mathbf{r}, E, \mathbf{\Omega}) \times \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \Sigma_s(\mathbf{r}, E' \to E, \mathbf{\Omega}' \to \mathbf{\Omega}, t) f(\mathbf{r}, E', \mathbf{\Omega}', t)$

  is "the total importance of the neutrons produced by collisions per unit time";

- $\widetilde{\beta}_i(t) := \dfrac{\mathcal{I}_{f,d,i}^0(t)}{\mathcal{I}_f^0(t)}$ is the "effective fraction of the i-th group delayed neutrons";

- $\mathcal{I}_{f,d,i}^0(t) := \int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \beta_i \nu \Sigma_f(\mathbf{r}, E, t) f(\mathbf{r}, E, \mathbf{\Omega}, t) \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \dfrac{\chi_{d,i}(\mathbf{r}, E')}{4\pi} \psi_0^*(\mathbf{r}, E', \mathbf{\Omega}')$

  is "the total importance of the delayed neutrons of the i-th group produced per unit time";

- $\widetilde{c}_i(t) := \dfrac{1}{\mathcal{I}^0(t)} \int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \dfrac{\chi_{d,i}(\mathbf{r}, E)}{4\pi} c_i(\mathbf{r}, t) \psi_0^*(\mathbf{r}, E, \mathbf{\Omega})$

  is the "effective concentration of the delayed neutron precursors of the i-th group".

On the other hand, the original delayed neutron concentration equations are preserved:

$$\frac{\partial c_i}{\partial t} - \frac{\nu_s}{\mathrm{Sc}} \Delta c_i + \nabla \cdot (c_i \mathbf{v}) + \lambda_i c_i = \int_{E_{\min}}^{E_{\max}} \mathrm{d}E' \int_{\mathbb{S}_2} \mathrm{d}\Omega' \, \beta_i \, \nu\Sigma_f(\mathbf{r}, E', t) \, a(t) \, f(\mathbf{r}, E', \mathbf{\Omega}', t) \, , \; 1 \leq i \leq F$$

⇒ The *spatio*-temporal concentrations c_i (eventually spatially 1-dimensional in system-scale codes) are solved for.

⇒ The effective concentrations used in the amplitude function are obtained by direct evaluation using their definition:

$$\widetilde{c}_i(t) := \frac{\int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \frac{\chi_{d,i}(\mathbf{r}, E)}{4\pi} c_i(\mathbf{r}, t) \psi_0^*(\mathbf{r}, E, \mathbf{\Omega})}{\int_V \mathrm{d}r \int_{E_{\min}}^{E_{\max}} \mathrm{d}E \int_{\mathbb{S}_2} \mathrm{d}\Omega \, \frac{1}{v} f(\mathbf{r}, E, \mathbf{\Omega}, t) \psi_0^*(\mathbf{r}, E, \mathbf{\Omega})}$$

(This model is similar to the "IPK" model of [Merle et al. 2015], inspired by the "Cinétique-point par zone" developed in [Laureau 2015].)

Work in progress, work to come

- Finalizing implementation in the system-scale code CATHARE.
- Verification and validation using the experimental data of the Aircraft Reactor Experiment.
- Application to support design and safety analyses of MSR reactors in CEA.

Fin.