Hybrid Data Assimilation methods: application to the DYNASTY experimental facility

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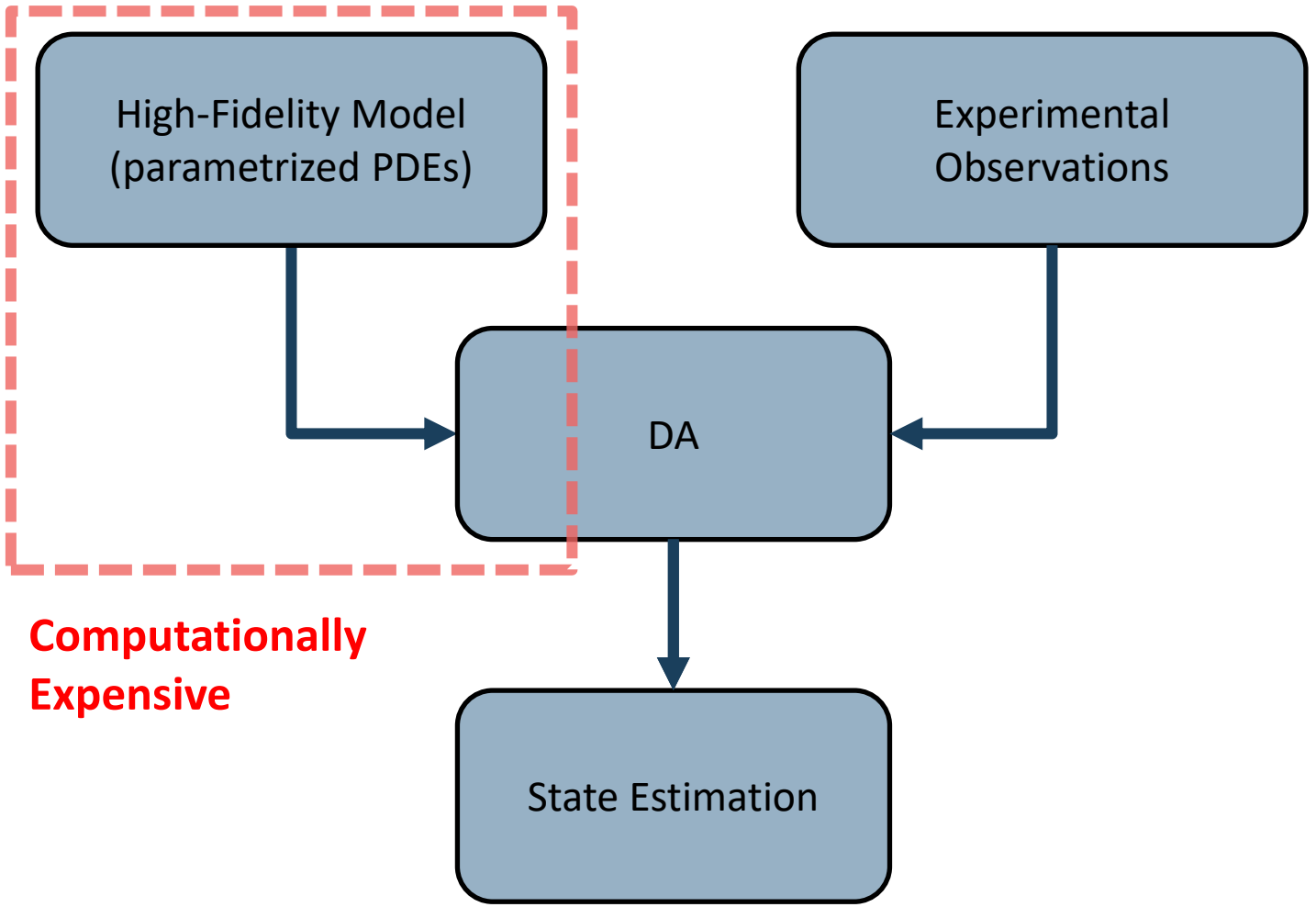
Data Assimilation (DA) framework



High-Fidelity models are solved using standard numerical methods, such as FE, FV, …

The computational cost is typically too heavy for **real-time** applications

Reduced Order Modelling,
via reduced basis methods

$$u(\mathbf{x};\ \mu) \simeq \sum_i c_i(\mu)\psi_i(\mathbf{x})$$

Basis Functions

Reduced Order Modelling (ROM) via reduced basis methods

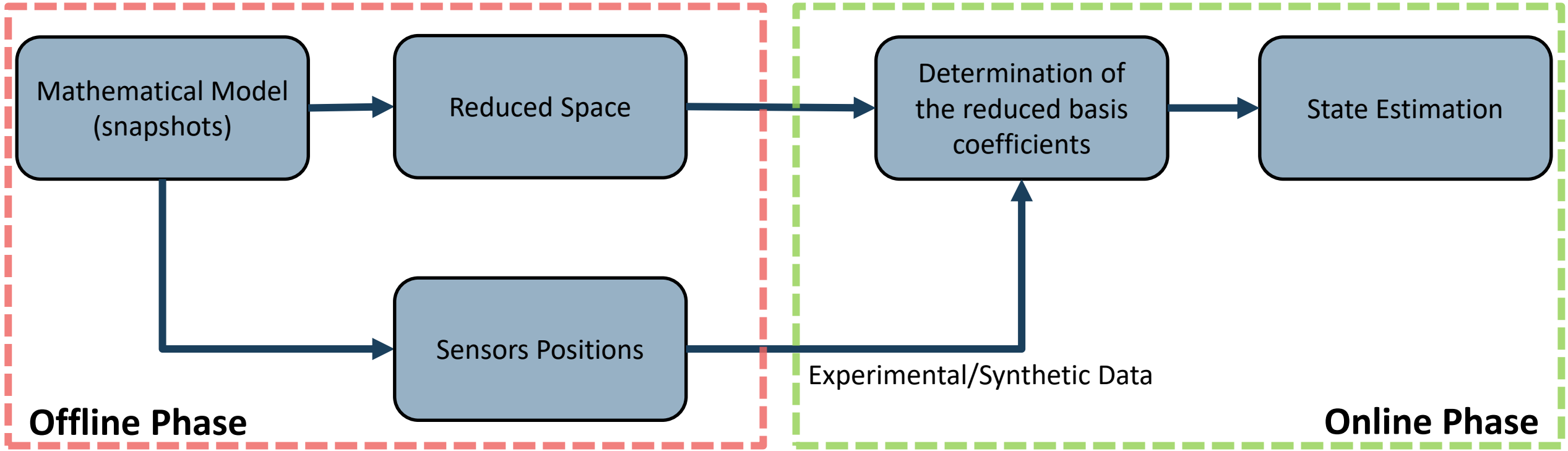
The solutions of the mathematical model can be approximated by an expansion on a **reduced basis**, able to catch the spatial behaviour of the functions

$$u(\mathbf{x};\ \mu) \simeq \sum_i c_i(\mu)\psi_i(\mathbf{x})$$

The generation of basis functions is typically computationally demanding, whereas the coefficients can be generated using two main approaches:

- **Intrusive**: a reduced order model is derived, from the high-fidelity one.
- **Non-intrusive**: the knowledge of the governing equations is not needed. Usually a linear system of small dimension is solved.

Hybrid Data Assimilation (HDA) methods



Generalized Empirical Interpolation Method (GEIM) & Parametrized-Background Data-Weak (PBDW) formulation

	GEIM	PBDW
Offline Phase	Generation of **Magic Functions** and **Magic Sensors**, with a greedy procedure	Generation of the basis functions for the **Reduced Space** and the **Update Space**
Online Phase	Collection of the experimental data, solution of a **lower triangular matrix** and **state estimation**	Collection of the experimental data, solution of an **almost full matrix** and **state estimation**

During the online phase, both algorithms impose an interpolation condition on the experimental data. Such condition can be weakened, in presence of **random noise**:

- GEIM: with the Tikhonov Regularization (TR-GEIM).
- PBDW: calibrating a suitable parameter . ξ

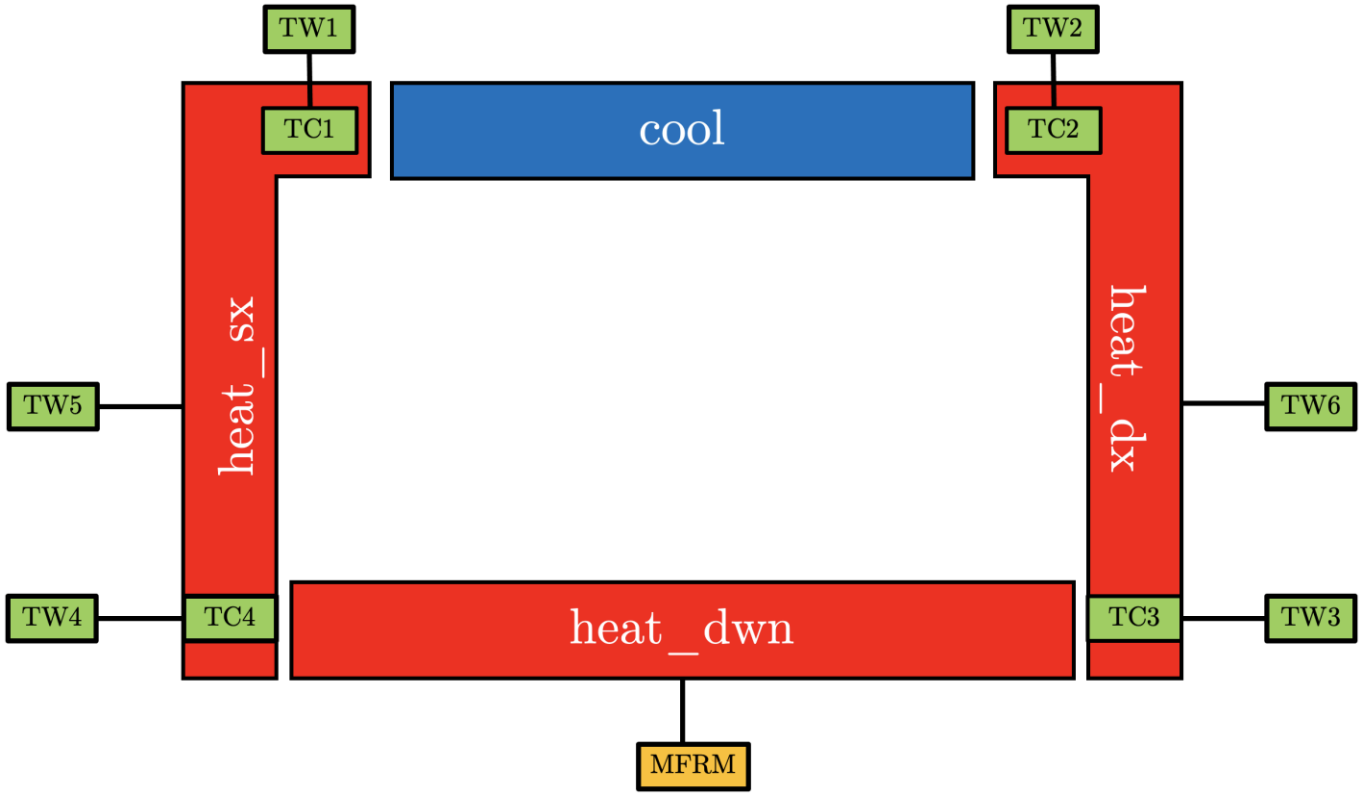
Validation case: DYNASTY facility

DYNASTY is an experimental facility, aimed to study the natural circulation regime and to simulate the primary circuit of Molten Salt Fast Reactors (MSFRs), with internal heat generation.

The fluid (water or molten salt) is heated through electrical stripes and cooled down with a fan.

Governing equations:

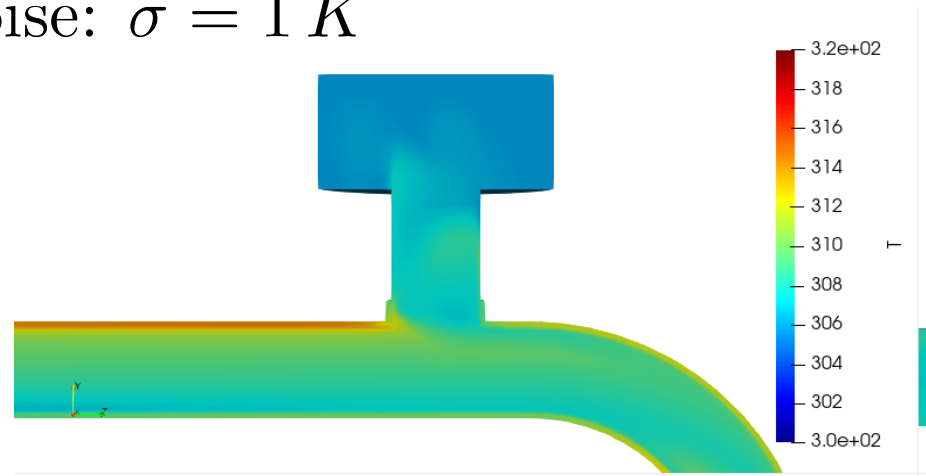
- *Fluid region:* Navier-Stokes equations
- *Solid region:* Heat diffusion equation



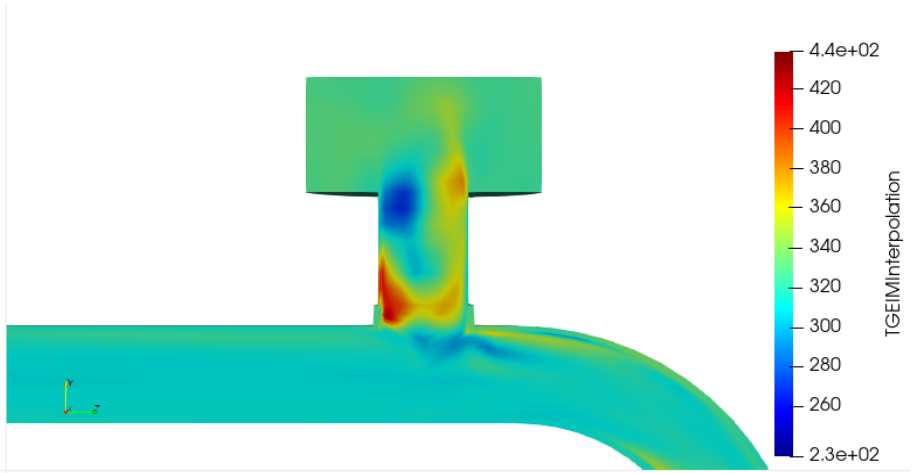
Synthetic Data: True Solution vs HDA

Random noise: $\sigma = 1\,K$

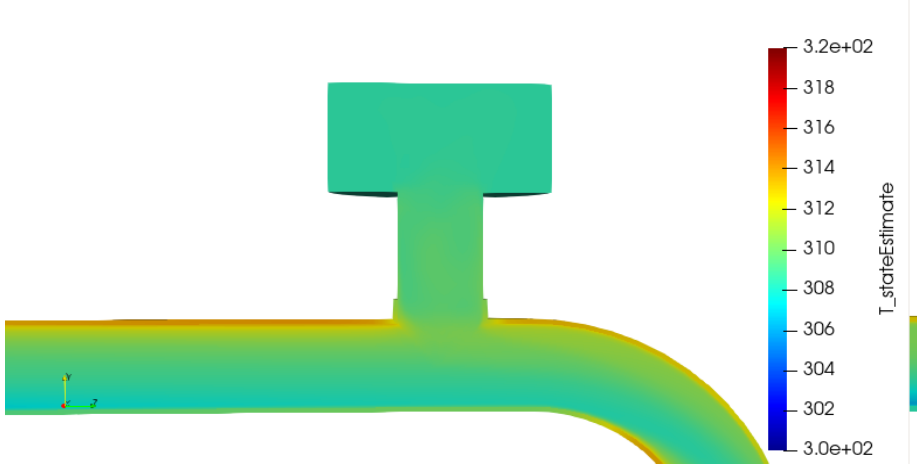
True Solution



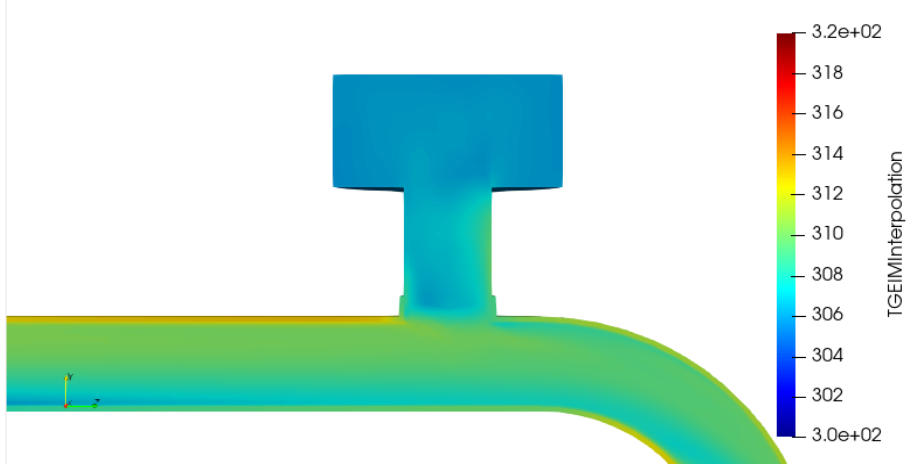
GEIM



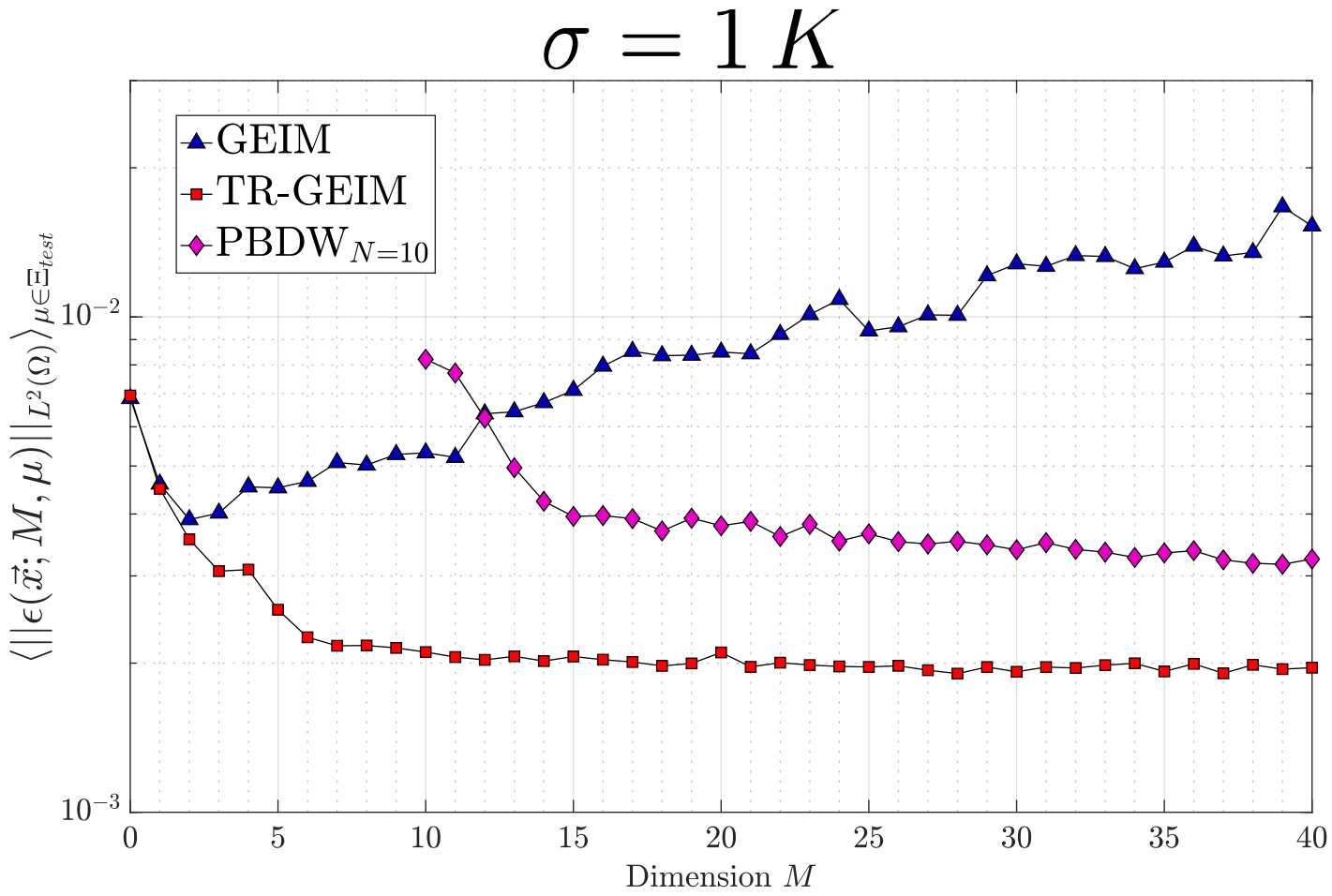
PBDW



TR-GEIM



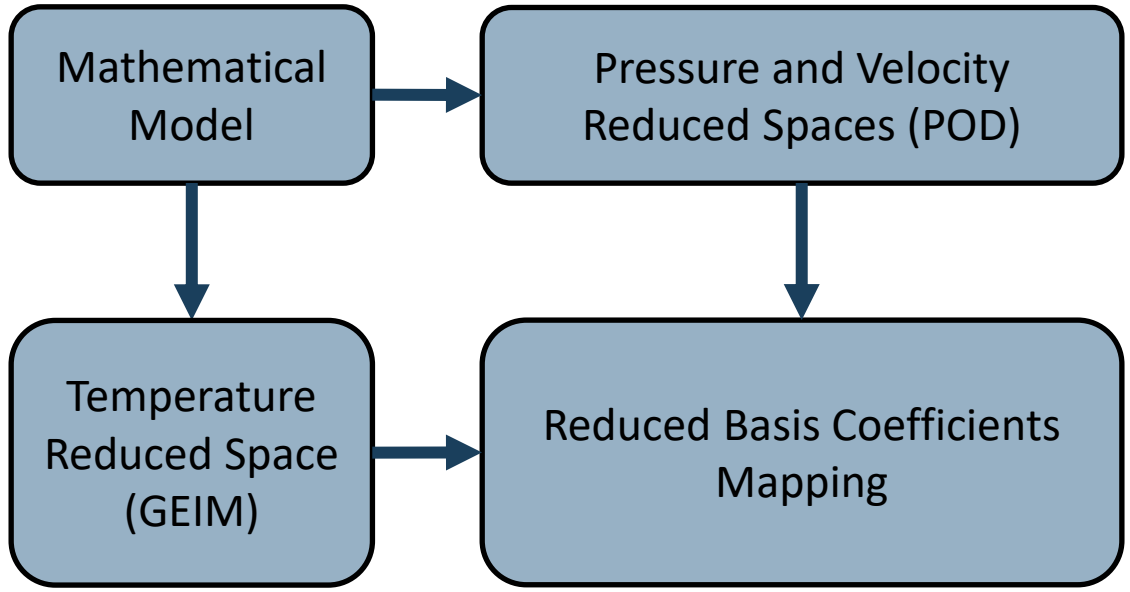
Online Phase: synthetic data



- **GEIM**: the method is clearly unstable
- **TR-GEIM**: the method is able to stabilize the procedure, providing optimal results in reconstructing the temperature field
- **PBDW**: the formulation provides almost optimal results, even though a lot of sensors may be required

Indirect Reconstruction (IR) algorithm

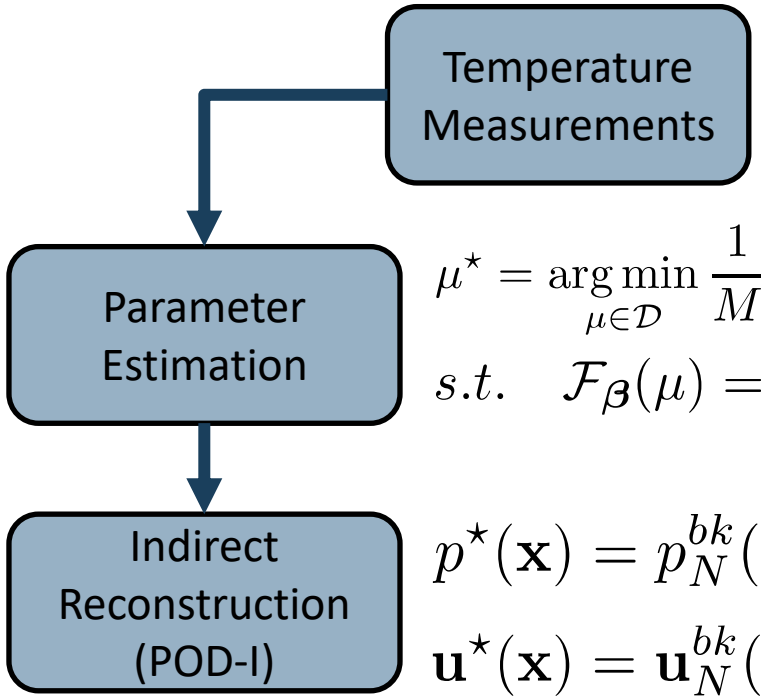
*Given **only temperature measurements available**, is it possible to estimate the **full state** of the system (e.g. pressure and velocity field)?*



$$\mathcal{F} : \mu \longrightarrow (\boldsymbol{\beta},\, \boldsymbol{\alpha}^{\mathbf{u}},\, \boldsymbol{\alpha}^{p})\,(\mu)$$

Offline Phase

Online Phase



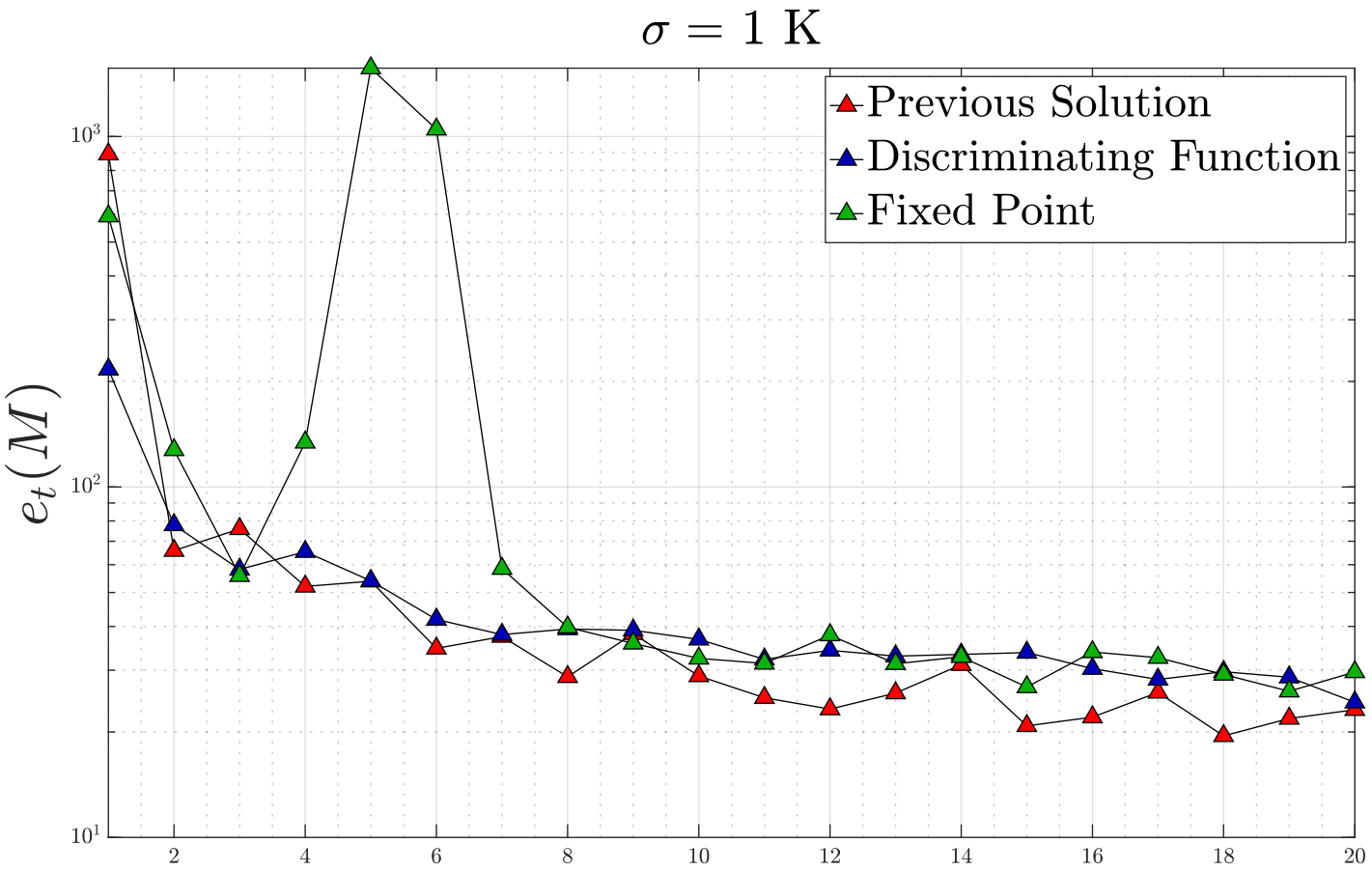
$$\mu^{\star} = \arg\min_{\mu \in \mathcal{D}} \frac{1}{M} \left|\left| B\boldsymbol{\beta}(\mu) - \mathbf{y}^{obs} \right|\right|_2^2$$

$$s.t. \quad \mathcal{F}_{\boldsymbol{\beta}}(\mu) = \boldsymbol{\beta}(\mu)$$

$$p^{\star}(\mathbf{x}) = p_N^{bk}(\mathbf{x};\, \mu^{\star})$$

$$\mathbf{u}^{\star}(\mathbf{x}) = \mathbf{u}_N^{bk}(\mathbf{x};\, \mu^{\star})$$

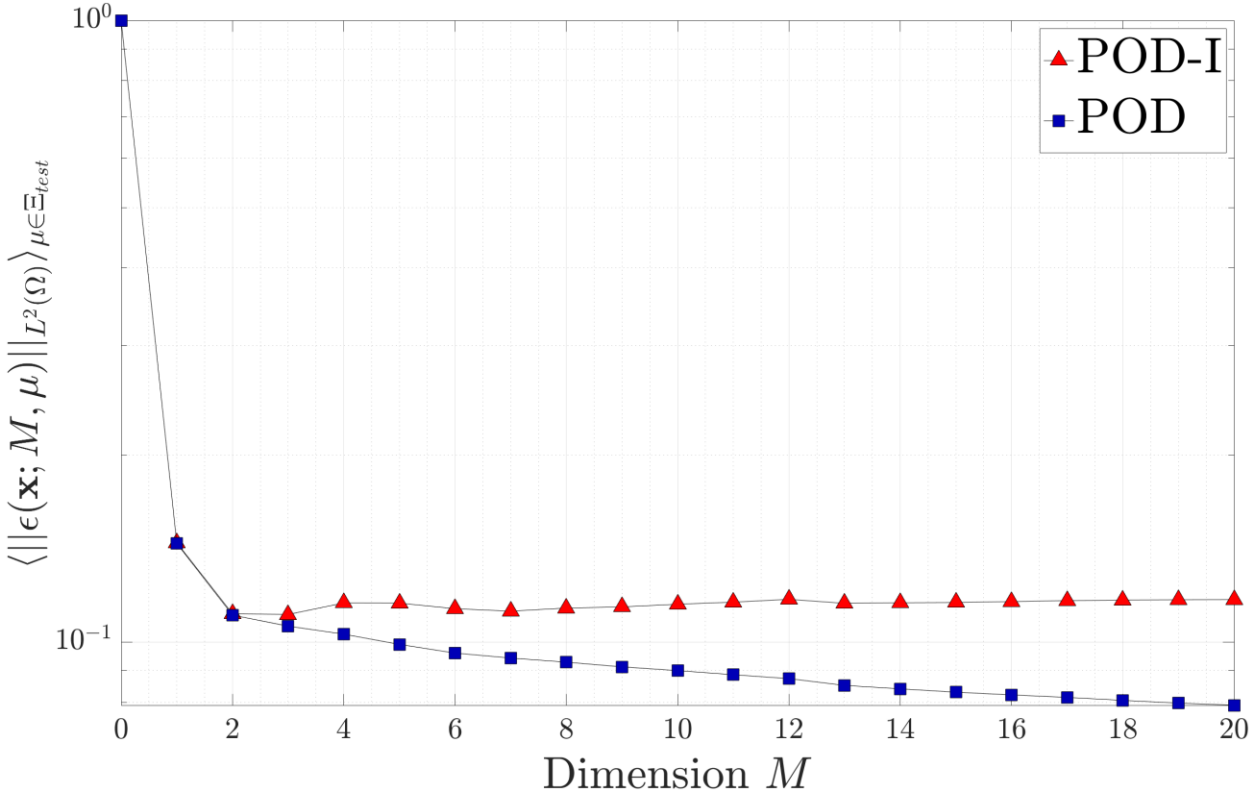
Parameter Estimation: effect of random noise – synthetic data



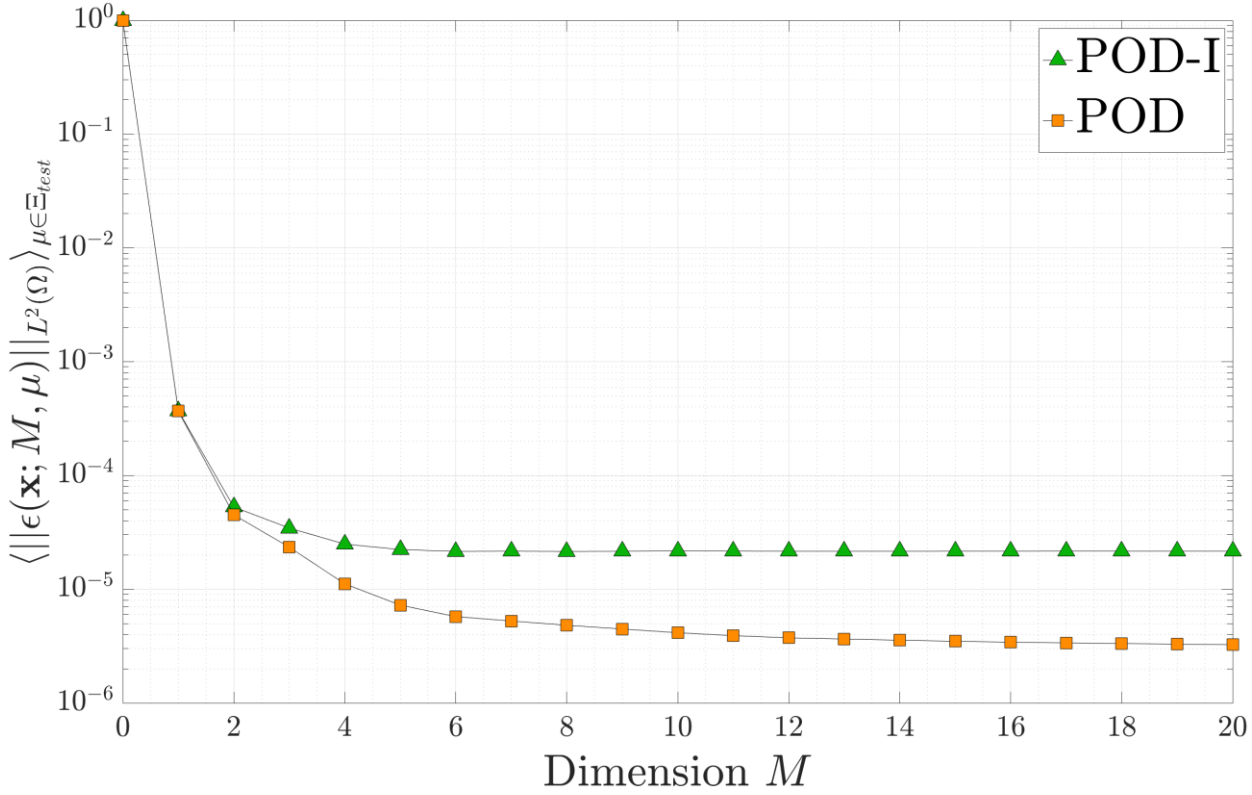
- The Discriminating Function strategy is really promising and its results are comparable with the Previous Solution one.
- The strong growth of the error is mainly due to the presence of local minima.
- None of the strategies suffers of instabilities, due to random noise.

POD with Interpolation – synthetic data

Velocity



Pressure



Conclusive remarks and future developments

(TR-)GEIM, PBDW and IR are promising tools for the integration of ROM methods in a DA framework. They can be applied to real complex experimental facilities.

Until now, the model has been considered perfect and only synthetic data have been considered:

- **If the model is inaccurate, how will these methods work?**
- **How will real experimental data impact the reconstruction through HDA?**